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An Enhancement and Comparison of Flexible Structure System Identification Methods for Control

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1. Introduction

The need for highly accurate pointing control of large flexible space structures has led to the development of many system identification techniques to provide models for modern control methods. Specifically, Q-Markov Covariance Equivalent Realization (COVER) and the Eigensystem Realization Algorithm (ERA) have received considerable attention. Both have been applied to large flexible structure testbeds such as the ACES facility at NASA Marshall [1], [2], [3], [4]. These methods were developed independently and little work has been done on comparing the two methods. In Section 2 of this paper, a brief description of the methods and theoretical comparisons are made. Emphasis is placed on the implications of the comparisons to the practical implementation of the methods.

While studying the two methods, it was noted that a quality measure, Degree of Modal Purity (DMP), used in ERA serves two key functions that were lacking in the Q-Markov COVER algorithm. One key function of system identification algorithms is that they provide not only a model of the system, but also information about the limitations/uncertainty in that model. This requirement is especially important when using robust control techniques such as H_∞, ℓ_1, or Maximum Entropy/Optimal Projection. A second key function of system identification algorithms is to eliminate spurious modes induced by noisy measurements in the identification experiment thus determining the proper system order. DMP provides these functions in ERA. In Section 3 of this paper, DMP is developed for the Q-Markov COVER algorithm.

Finally, the two methods are applied to a two degree-of-freedom spring-mass-damper system to provide a demonstration of some of the method comparisons and the usefulness of the newly developed DMP for the Q-Markov COVER algorithm. Further method comparisons and implementation issues are also discussed. A summary of the application is given in Section 4.

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2. Method Comparisons

It is assumed that the structure to be identified can be represented as a discrete-time linear, time-invariant, finite dimensional system of the form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{2.1}$$

where $x \in \Re^{n_x}$, $y \in \Re^{n_y}$, $u \in \Re^{n_u}$. The system identification methods Q-Markov COVER and ERA are based on the concept of obtaining a discrete-time state space realization of a structure from time domain test data. Concepts from system/control theory are used to produce the realizations. Detailed description of the algorithms can be found in [5], [6], [7], and [8]. In the Q-Markov COVER algorithm, the matrix

$$D_q = O_q X O_q^* \tag{2.2}$$

is formed from pulse response data (Markov parameters) obtained from the system. In Equation 2.2 O_q and X are given by

$$\begin{gathered} O_q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{q-1} \end{bmatrix} \\ X = AXA^* + BWB^* = \begin{bmatrix} B & AB & A^2B & \ldots \end{bmatrix} * \mathrm{diag}(W,\ W,\ \ldots) * \begin{bmatrix} B^* \\ B^*A^* \\ B^*(A^2)^* \\ \vdots \end{bmatrix} \end{gathered} \tag{2.3}$$

In Equation 2.3 W is a diagonal matrix with the square of the input pulse magnitudes along the diagonal, and X is the state covariance matrix. A singular value decomposition is performed on D_q to determine the principle gains and directions. The resulting direction and gain matrices along with another matrix composed of pulse response data (Markov parameters) are used to compute the state space realization of the system. In the absence of noise, the first q Markov parameters and the first q covariance parameters of the identified system are guaranteed to be identical to those of the true system. In ERA the generalized Hankel matrix

$$H_{\xi\eta}(k) = V_\xi A^k W_\eta \tag{2.4}$$

is formed from pulse response data (Markov parameters) obtained from the system. In Equation 2.4 V_ξ, a generalized observability matrix, and W_η, a generalized controllability matrix, are given by Equation 2.5.

$$V_\xi = \begin{bmatrix} C \\ C_{J_1}A^{s_1} \\ \vdots \\ C_{J_\xi}A^{s_\xi} \end{bmatrix} \qquad W_\eta = \begin{bmatrix} B & A^{t_1}B_{K_1} & \dots & A^{t_\eta}B_{K_\eta} \end{bmatrix} \tag{2.5}$$

In Equation 2.5 s_i, t_i are used to select data from particular times and J_i, K_i are used to select data from particular outputs/inputs respectively. A singular value decomposition is performed on the generalized Hankel matrix with k = 0. The resulting direction and gain matrices along with a shifted generalized Hankel matrix ($k \neq 0$) are used to compute the state space realization of the system. In both algorithms noisy data is handled by truncating the number of singular values used in calculating the state space realization.

As can be discerned from the brief descriptions, several concepts are common to the two methods. For example, both use Markov parameters as structural data and both use a singular value decomposition to break the data into its principle components. One can also see that the concepts of observability and controllability from system/control theory are highly prevalent in both methodologies. As a result of this last similarity, the identified model will contain more accurate information on the state, input, and output relationships of the structure than models obtained from standard modal analysis methods. Since many modern control methods rely on the accuracy of these relationships, the use of these identification algorithms is desirable. Finally, it can be shown (proof available) that Q-Markov COVER and ERA produce equivalent realization when the parameters of ERA are chosen to be $s_i = (1,...q)$, $t_i = (1,...,\infty)$, $J_i = (1,...,n_y)$, $K_i = (1,...,n_w)$, and shifted k = 1. This equivalence, however, does not hold when singular values are truncated to reduce the effect of noise.

When the parameters are not chosen as discussed previously, there are differences between the methods. One major difference between the two algorithms is that ERA does not necessarily incorporate covariance information. The covariance is a function of the long term time domain behavior of the system as are the damping and low frequency gain; thus, the lack of covariance information results in damping biases and low frequency gain errors in the resulting identified systems. On the other hand, calculating the covariance is computationally expensive and time consuming; thus, there are benefits to not including it. Another major difference between the algorithms is the flexibility that ERA allows in choosing data with the parameters s, t, K, and J. This allows one to eliminate particularly noisy data from the identification process; yet still identify the system.

Recently, two new identification methods have been developed based on ERA and Q-Markov COVER. The first method, Eigensystem Realization with Data Correlations (ERA/DC) [9] incorporates correlation information into the ERA method. The second method, Observability Range Subspace Extraction (ORSE) [10] is an extension of the Q-Markov COVER algorithm that produces an identified model from colored noise response data. While both of these methods expand the scope of the methods on which they are based, the expense is an increase in computational complexity.

3. Development of Degree of Modal Purity for Q-Markov COVER

The concept of Degree of Modal Purity (DMP) was initially developed for the Eigensystem Realization Algorithm in [5] and [6]. It is a method for determining the accuracy of each mode in the identified model. Accuracy is determined by calculating the coherence between the extrapolated modal time histories from the identified model and modal components of the time domain pulse response data. This information is used to find and truncate spurious modes produced by noisy data and to provide an accuracy measure for each mode of the identified system. Previously, there was no such tool to provide these functions for the Q-Markov COVER algorithm.

From the singular value decomposition of D_q it is possible to write

$$D_q = P_q P_q^* \tag{3.1}$$

From Equations 2.2 and 2.3, it is clear that P_q is simply the observability-like matrix for a state space representation of the structure with unitary state covariance. Let $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$ be the state space representation of the discrete-time identified system obtained directly from the Q-Markov COVER procedure. Let p_i, ψ_i be the eigenvalues and eigenvectors of $\hat{A}$ respectively. Form the modal representation, A_m, B_m, C_m, D_m, of the system by using a transformation matrix whose columns are ψ_i, the eigenvectors of $\hat{A}$.

$$A_m = \mathrm{diag}(p_1, p_2, \ldots p_n) \qquad B_m = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \qquad C_m = \begin{bmatrix} c_1 & \cdots & c_n \end{bmatrix} \tag{3.2}$$

For each mode of the identified system, form the extrapolated observability-like modal time history

$$\bar{r}_i = \begin{bmatrix} c_i^* & c_i^* p_i & \cdots & c_i^* p_i^{(Q-1)} \end{bmatrix}^* \tag{3.3}$$

Project the modal time histories of the data using the corresponding eigenvector

$$r_i = P_q * \psi_i \tag{3.4}$$

If the i[th] mode is a true linear mode of the system, then $\bar{r}_i$ and r_i will be collinear. The value of γ_i given by

$$\gamma_i = \frac{r_i^* \bar{r}_i}{|r_i^* r_i|\,|\bar{r}_i^* \bar{r}_i|} \tag{3.5}$$

will be very close to 1 for true linear system modes. Noise modes, inaccurately identified modes, and modes induced by strong nonlinearities will have values of γ_i less than 1. Note that the value of γ_i quantifies the degree to which the i[th] mode is observed in the output.

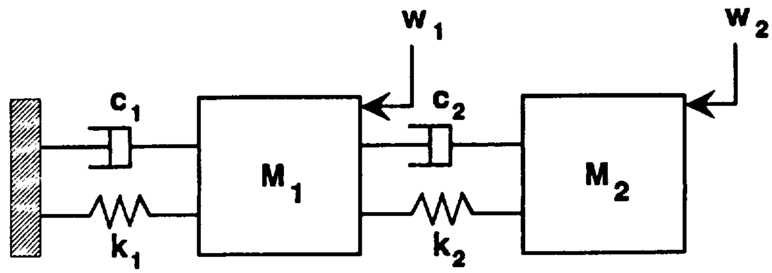

Figure 4.1 Spring-Mass-Damper System

4. Identification of a Two Degree-of-Freedom System

4.1 System Description

In order to demonstrate some of the method comparisons and the usefulness of the Degree of Modal Purity for Q-Markov COVER, the two methods discussed in this paper are applied to a two degree-of-freedom, two input, two output, spring-mass-damper system. The inputs are the forces on the two masses, and the outputs are the position of the two masses. The state space representation of the continuous time system is given by

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cccc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -15 & 7.5 & -0.4 & 0.2 & 0 & -0.25 \\ 30 & -30 & 0.8 & -0.8 & -1 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \tag{4.1}$$

The continuous time modes of the system are described in Table 4.1. The system was designed to have one long slow mode (Mode 1) and one short fast mode (Mode 2). The data used for the identification were the output responses to pulses of height 50 sampled at a rate of 10 Hz. Zero mean white noise with variance 0.01 was injected at the sensors producing a low signal to noise ratio. The continuous time response to a pulse of height 50 width 0.1 without noise and the noisy sampled response to the same input are shown in Figures 4.2 and 4.3.

Mode #	Frequency	Damping
1	0.9939 Hz	0.0836
2	0.3808 Hz	0.0319

Table 4.1 Modal Data

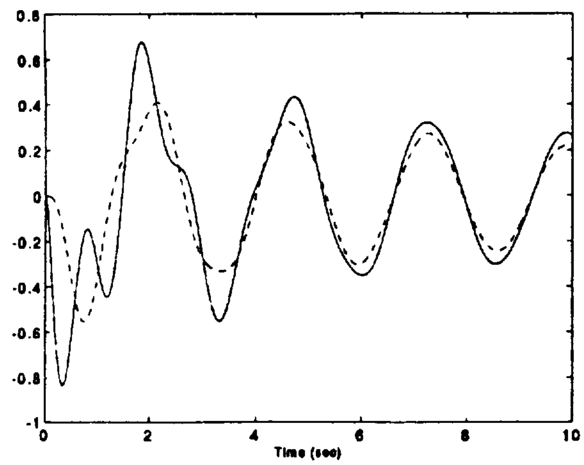

Figure 4.2 Continuous Time Response to Pulse on Input 1 (Width 0.1, Height 50)

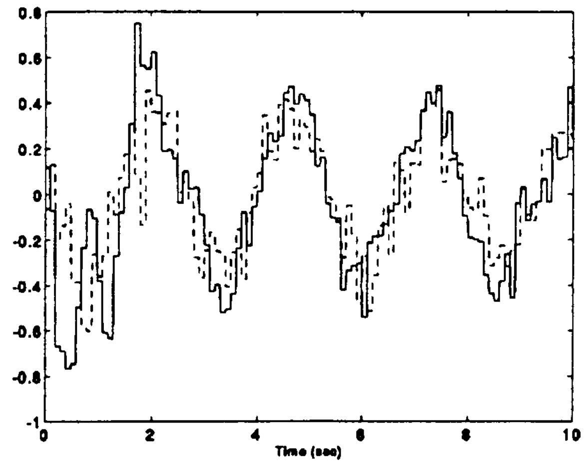

Figure 4.3 Noisy Sampled Response to Pulse on Input 1, (Width 0.1, Height 50)

4.2 Choice of Parameters

In each method, parameters specifying the amount of data to be used by the method must be chosen. The parameter q in Q-Markov COVER determines the number of Markov and covariance parameters of the identified systems to be matched with the data from the true system. It also determines the dimension of D_q and the maximum order of the identified system. The parameters ξ and η determine the dimensions of the Hankel matrix used in ERA with ξ being the maximum order of the identified system. As discussed in Section 2, other parameters (s, t, J, K) are available with ERA to reduce the data in the Hankel matrix by eliminating data from certain times, inputs or outputs. This reduction of data is not used in this example.

Tables 4.2 and 4.3 show the results of varying method parameters on the eigenvalues of the discrete-time identified system. Both methods produced accurate identified models. Data from the final models is shown in the highlighted rows of Tables 4.2 and 4.3 and Figures 4.4 and 4.5. Relating these results back to the discussion in Section 2, the damping error for the ERA identified models is generally larger than for the Q-Markov COVER identified models. Note that the accuracy of the identified model is much more sensitive to parameter variations for ERA than for Q-Markov. Including covariance information in Q-Markov COVER has an averaging effect that reduces the sensitivity to the parameter q.

For both methods, it is clear that there is an optimal choice of parameters. When the parameters are chosen too large, noise effects decrease accuracy. Due to the natural damping of the structure, the amplitude of the pulse response of the system will decay while the magnitude of the sensor noise will remain constant thus decreasing the signal to noise ratio with time. When parameters are increased, the additional data has a lower signal to noise ratio than previous data. When enough of this noisy data is included, the identification results become corrupted. This effect is especially prevalent in Mode 2 which dies out more quickly than Mode 1. When the parameters are chosen too small, not enough data is included to properly identify the modes. As general guidelines one should choose parameters to include data from a full period of the lowest frequency mode, while not including data long past when the signal to noise ratio of major modes has significantly degraded. If these guidelines conflict, decisions must be made as to modal priority. Comparing frequency responses of the identified system with test data is also beneficial in fine tuning the choice of parameters.

ζ,η	Order	eigenvalues of $\hat{A}$ (discrete-time)	% error frequency	% error damping	Pulse Response Error*
10	4	0.9600 ± 0.2290i	2.01	-75.97	4.5753
		0.7203 ± 0.5883i	-9.90	-26.16	
20	4	0.9537 ± 0.2410i	-3.65	-107.47	4.9880
		0.7658 ± 0.5578i	-0.83	-2.26	
29	4	**0.9636 ± 0.2360i**	**-0.39**	**-3.47**	**0.8992**
		0.7603 ± 0.5595i	**-1.66**	**-8.29**	
40	4	0.9679 ± 0.2369i	-0.31	53.42	2.9177
		0.7589 ± 0.5581i	-1.64	-12.28	
50	4	0.9688 ± 0.2358i	0.02	36.51	1.8151
		0.7496 ± 0.5556i	-2.38	-29.32	

Table 4.2 Choice of Parameters for Eigensystem Realization Algorithm Identification

Q	Order	eigenvalues	% error frequency	% error damping	Pulse Response Error *
10	4	0.9643 ± 0.2339i	0.52	-2.22	1.7726
		0.7676 ± 0.5399i	1.67	-23.33	
20	4	0.9639 ± 0.2359i	-0.31	-0.87	0.7754
		0.7735 ± 0.5529i	0.64	2.97	
30	4	**0.9645 ± 0.2350i**	**0.11**	**4.26**	**0.7118**
		0.7676 ± 0.5558i	**-0.38**	**-2.29**	
40	4	0.9649 ± 0.2353i	0.03	10.32	0.8777
		0.7616 ± 0.5553i	-0.97	-11.85	
50	4	0.9646 ± 0.2350i	0.14	5.93	1.0491
		0.7457 ± 0.5546i	-2.71	-36.26	

Table 4.3 Choice of Parameters for Q-Markov COVER Identification

* The Pulse Response Error was calculated as the sum of the ℓ_2 norm of the difference between the identified and true discrete time pulse response for 20 seconds.

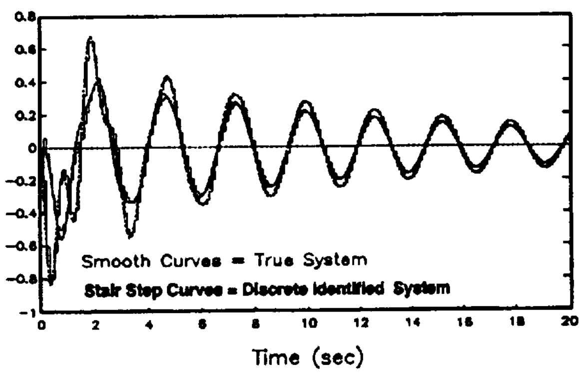

Figure 4.4 Pulse Response (Input 1) of Q-Markov COVER Identified System

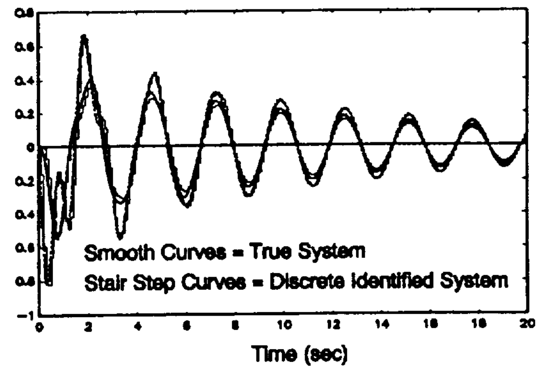

Figure 4.5 Pulse Response (Input 1) of ERA Identified System

4.3 Use of Degree of Modal Purity with Q-Markov COVER

The usefulness of DMP with ERA has been demonstrated in [5] and [6]. It is shown in this paper that DMP can provide similar benefits for Q-Markov COVER. Table 3.1 shows the results of the identification using Q-Markov COVER with DMP for different choices of system order. When the model order is made larger than 4, the DMP for the spurious modes decreases indicating that the mode is not a true system mode. Note that the value of the DMP for Mode 2 is always lower than for Mode 1. Mode 1 is the dominant mode, so it is reasonable that identification for Mode 2 would be slightly less accurate. This information could be incorporated into norm based methods such as H_∞ and ℓ_1 as uncertainty via frequency domain weighting. The modal uncertainty could also be incorporated into the Maximum Entropy/Optimal Projection method via parametric uncertainty in the A matrix. Similar results were obtained using ERA with DMP.

Q	Order	Mode #	eigenvalues (discrete system)	DMP	% error frequency	% error damping
30	2	1	$0.9646 \pm 0.2345i$	0.9998	0.30	3.31
30	4	1	$0.9639 \pm 0.2350i$	0.9999	-0.11	4.26
		2	$0.7676 \pm 0.5558i$	0.9831	-0.38	-2.29
30	6	1	$0.9645 \pm 0.2350i$	0.9999	0.11	4.55
		2	$0.7746 \pm 0.5561i$	0.9827	0.36	8.85
		-	$-0.3724 \pm 0.8914i$	0.9137	-	-
30	8	1	$0.9646 \pm 0.2348i$	0.9999	-0.23	-0.21
		2	$0.7801 \pm 0.5537i$	0.9888	1.15	10.80
		-	$-0.3718 \pm 0.8914i$	0.9136	-	-
		-	$0.4101 \pm 0.5429i$	0.5088	-	-

Table 4.4 Use of DMP for Identification of Example

5. Conclusion

This paper provides needed information on two popular techniques for structural identification, Q-Markov COVER and the Eigensystem Realization Algorithm. Theoretical comparisons based on system/control theory were related to practical implementation issues. Comparisons of the two methods show that it is possible to choose parameters for ERA so that the method is equivalent to Q-Markov COVER. Other parameter choices, however, result in the methods behaving very differently. The adaptation of the Degree of Modal Purity to the Q-Markov COVER Algorithm provides the method with two key attributes previously lacking in the method. Quantitative information on the quality of the identified modes is now available for use in determining system order from noisy data. This measure also provides the level of uncertainty of each mode that is useful when applying robust control methods. The methods were applied to a simple two-degree-of-freedom system for demonstration purposes. Guidelines for choosing parameters in each method were presented. Work is currently in progress to apply the methods to the Controls, Astrophysics, and Structures Experiment in Space (CASES) at the NASA Marshall Space Flight Center [11].

6. References

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